NOTATION

$T(x, t)$, temperature at point x; t, times; $C_S(T)$, $\lambda_S(T)$, heat capacity and thermal conductivity coefficients; α, heat exchange coefficient; T_0, temperature of external medium; $Q(t)$, specific power of heat sources.

LITERATURE CITED

1. A. T. Pugachev, Yu. A. Volkov, and N. P. Churakova, Inzh. Fiz. Zh., 38, No. 4, 606-613 (1980).
2. A. T. Pugachev and N. P. Churakova, Zav. Lab., 46, No. 8, 737-738 (1980).
3. A. T. Pugachev, S. I. Kovalenko, N. P. Churakova, et al., Fiz. Tverd. Tela, 46, No. 8, 2468-2471 (1981).
4. O. M. Alifanov, Converse Heat Exchange Problems [in Russian], Moscow (1988).
5. O. M. Alifanov and A. P. Tryanin, Inzh. Fiz. Zh., 48, No. 3, 472-473 (1985).
6. E. A. Artyukhin, Inzh. Fiz. Zh., 48, No. 3, 490-495 (1985).
7. E. A. Artyukhin, Inzh. Fiz. Zh., 41, No. 4, 587-592 (1981).
8. E. A. Artyukhin and A. S. Okhapkin, Inzh. Fiz. Zh., 45, No. 5, 781-788 (1983).
9. N. V. Muzylev, Zh. Vychisl. Mat. Matem. Fiz., 20, No. 2, 388-400 (1980).
10. Yu. V. Tret'yakov, Fizich. Élektron., No. 14, 91-92 (1977).
11. A. V. Lykov, Thermal Conductivity Theory [in Russian], Moscow (1967).
12. G. I. Marchuk and V. I. Agoshkov, Introduction to Projection Grid Methods [in Russian], Moscow (1981).
13. F. P. Vasil'ev, Methods for Solution of Extremal Problems [in Russian], Moscow (1981).
14. G. Douglas, G. Dupont, and R. S. Ewing, J. Numer. Anal., 19, No. 3, 503-522 (1979).

DETERMINATION OF THERMAL CONTACT RESISTANCES USING THE SPECTRAL FUNCTIONS OF BOUNDARY EFFECTS

O. S. Tsakanyan

UDC 536.24

A method is proposed for determining thermal contact resistances by solving the inverse heat-conduction problem.

The growing requirements for the design of economical heat machines cannot be satisfied without knowledge of the heat processes occurring in them. The study of heat-exchange processes involves a full-scale thermophysical experiment which provides information about the temperature field from a limited set of observation points, located inside the machine; this information is then used to solve the inverse heat-conduction (IHC) problem in order to find the boundary effects, which are necessary for determining the thermally stressed state of the parts and units of heat machines.

It is of particular interest to determine boundary conditions of the fourth kind, i.e., the thermal contact resistances (TCR's) between the surfaces of the parts in contact, with the aid of the solution of the IHC problem from the results of a thermophysical experiment.

The dynamics of the thermal process for a composite body is described by the heat equation

$$\frac{\partial}{\partial x}\left[\lambda(x,\ y,\ T)\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda(x,\ y,\ T)\frac{\partial T}{\partial y}\right]=c_V(x,\ y,\ T)\frac{\partial T}{\partial t}. \quad (1)$$

Besides Eq. (1), the mathematical model (of the phenomenon under consideration) determining the thermal state of the object also contains the initial edge conditions

Institute of Problems in Mechanical Engineering, Academy of Sciences of the Ukrainian SSR, Khar'kov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 58, No. 1, pp. 135-140, January, 1990. Original article submitted September 22, 1989.

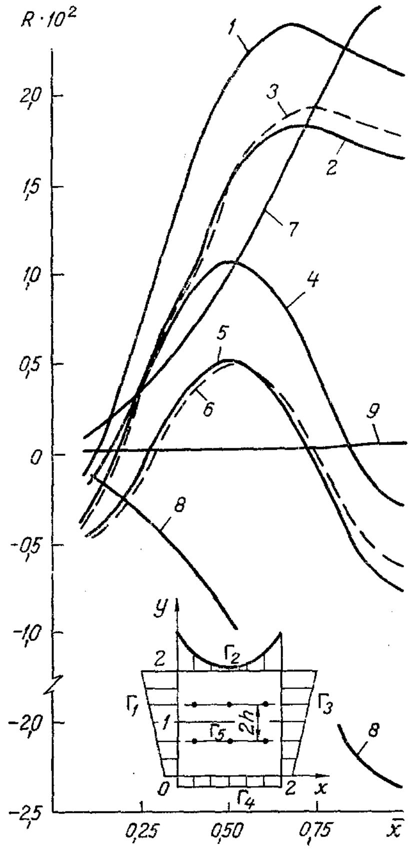

Fig. 1. Thermal contact resistances obtained by solving the IHC problem.

$$T = T(x, y), \ t = 0 \tag{2}$$

and the boundary conditions

of the first kind: $T_{sur} = T(x, y, t)$, (3)

of the second kind: $q_{sur} = q(x, y, t)$,

of the third kind: $\alpha(x, y, t, T)(T_{sur} - T_m) = -\lambda(x, y, T)\frac{\partial T}{\partial n}$, (4)

of the fourth kind; in the case of an ideal contact

$$T_{1sur} = T_{2sur}, \tag{5}$$

$$-\lambda_1(T_1)\frac{\partial T_1}{\partial n}\Big|_{sur} = -\lambda_2(T_2)\frac{\partial T_2}{\partial n}\Big|_{sur}, \tag{6}$$

and in the case of a nonideal contact

$$\frac{1}{R(x, y, t)}(T_{1sur} - T_{2sur}) = -\lambda_1(T_1)\frac{\partial T_1}{\partial n}\Big|_{sur}, \tag{7}$$

$$= -\lambda_1(T_1)\frac{\partial T_1}{\partial n}\Big|_{sur} = -\lambda_2(T_2)\frac{\partial T_2}{\partial n}\Big|_{sur}. \tag{8}$$

In order to solve the IHC problem we must know the temperature dependences for the observation points, which are obtained from a physical experiment

$$T_s = T(x_s, y_s, t), \ s = 1, 2, \ldots, N, \tag{9}$$

where N is the number of observation points.

Let us consider the method of solution on the example of contact between two regions in the form of rectangles with sides in a 2:1 ratio. Heat conduction for regions Ω_1 and Ω_2 is described by the equation

$$\nabla^2 T = 0. \tag{10}$$

Boundary conditions of the first kind are given on boundary segments Γ_1, Γ_2, Γ_3, and Γ_4 (see Fig. 1) in the form of the polynomials

$$T_i(x, y) = \sum_{j=0}^{m_i} a_{ij} y^j \begin{cases} i = 1, & x = 0, \ 0 \leqslant y \leqslant 2; \\ i = 3, & x = 2, \ 0 \leqslant y \leqslant 2; \end{cases}$$

$$T_i(x, y) = \sum_{j=0}^{m_i} a_{ij} x^j \begin{cases} i = 2, & 0 \leqslant x \leqslant 2, \ y = 2; \\ i = 4, & 0 \leqslant x \leqslant 2, \ y = 0. \end{cases} \quad (11)$$

We must determine the TCR at boundary Γ_5 from the known values of the temperature at observation points on either side of boundary Γ_5 at a distance of 0.1ℓ, where ℓ is the width of a rectangular element.

To calculate the TCR's we must determine the surface temperatures $T_1(x, 1)$ and $T_2(x, 1)$ at Γ_5 for both surfaces of regions Ω_1 and Ω_2 and the heat flux $q_5(x, 1)$, passing through the contact boundary, after which we calculate from the formula

$$R(x, 1) = \frac{T_1(x, 1) - T_2(x, 1)}{q_5(x, 1)}. \quad (12)$$

The heat flux $q_5(x, 1)$ acting on boundary Γ_5 is found as a polynomial

$$q_5(x, 1) = \sum_{j=0}^{m_i} b_{5j} x^j, \ 0 \leqslant x \leqslant 2,$$

in which the parameters b_{5j} of the boundary effects are unknown. Functions x^j are called the spectral components of the boundary effects (any continuous functions, e.g., $\sin(jx)$ or $\exp(\lambda_j x)$, can be taken to be the spectral components of the boundary effect in the general case).

If we apply the j-th spectral component of the boundary effect to a segment of the boundary, e.g., Γ_5, while applying zero boundary effects to the remaining segments in accordance with the kind of boundary conditions assigned to them, and then solve the heat problem, we obtain the spectral function $W_{ij}(x, y)$ of the given boundary effect [1]. The complete solution of the stationary heat-conduction problem in linear formulation can be represented as the superposition of the products of the boundary-effect parameters and the corresponding spectral function:

$$T(x, y) = \sum_{i=1}^{n} \sum_{j=0}^{m_i} a_{ij} W_{ij}(x, y). \quad (13)$$

For the nonstationary heat-conduction problem the solution in the k-th time interval (when the implicit finite-difference approximation scheme is applied to the heat equation) can be represented as

$$T^{(k)}(x, y) = \sum_{i=1}^{n} \sum_{j=0}^{m_i} a_{ij}^{(k)} W_{ij}(x, y) + T_n^{(k)}(x, y), \quad (14)$$

where the first term is the reaction to the boundary effect in the k-th time interval and the second term, $T_n^{(k)}(x, y)$, is the reaction to the temperature field $T^{(k-1)}(x, y)$.

When solving the linear heat-conduction problem with a constant time interval we determine the spectral functions only once before the problem is actually solved. the situation changes when the problem is in the nonlinear formulation, since an iteration process is organized to refine the thermal conductivity and capacity, which depend on the sought-after function of the temperature. In this case before each interaction the spectral functions are determined with the thermophysical properties of the object "frozen" with respect to the thermal state obtained in the previous iteration. It is pointless to use such spectral functions to solve linear and nonlinear problems in the direct formulation since this requires considerable computer time and additional memory to store the files of the functions.

In regard to the IHC problem and the problem of optimization of thermal process it is convenient to factor the boundary effects into components and to determine the spectral functions during the analysis of the thermal state of the object.

The boundary-effect parameters b_{5j} are identified by solving the system of linear algebraic equations

$$\sum_{j=0}^{m_i} b_{5j} W_{5j}(x_s,\ 1+h)\big|_{\Omega_1} = T_1^{(k)}(x_s,\ 1+h) - T_{\Omega_1}^{(k)}(x_s,\ 1+h),\ s=1,\ \ldots,\ N_1; \tag{15}$$

$$\sum_{j=0}^{m_i} b_{5j} W_{5j}(x_s,\ 1-h)\big|_{\Omega_2} = T_2^{(k)}(x_s,\ 1-h) - T_{\Omega_2}^{(k)}(x_s,\ 1-h),\ s=1,\ \ldots,\ N_2. \tag{16}$$

Here $T_1^{(k)}(x_s,\ y_s)$ and $T_2^{(k)}(x_s,\ y_s)$ are the temperatures at the observation points and $T_{\Omega 1}^{(k)}(x_s,\ y_s)$ and $T_{\Omega 2}^{(k)}(x_s,\ y_s)$ are the reaction to the known boundary effects and the temperature field $T^{(k-1)}(x,\ y)$.

In the mathematical treatment of the results of thermophysical experiments we may come across different variants of the arrangement of observation points, which necessitates some changes in the computational algorithm. For example, the observation points may lie in one region Ω_1 at a distance $h = 0.1\ell$ from boundary Γ_5 (Fig. 1) or in both regions Ω_1 and Ω_2 at a distance h on either side of boundary Γ_5. In the first case the parameters b_{5j} are determined from the solution of system (15) while in the second case they are found from the solution of systems (15) and (16).

In the case of an indeterminate system of equations we use the method of least squares, which enables us to symmetrize the matrix of the initial system of equations and thus obtain the solution.

The surface temperatures $T_1(x,\ y)$ and $T_2(x,\ y)$ are found from the solution of the heat-con conduction problem for regions Ω_1 and Ω_2 with the boundary effects at Γ_1, Γ_2, Γ_3, and Γ_4 which are known from the formulation of the problem, and the heat flux $q_5(x,\ 1)$, which we found. After this the thermal contact resistances are calculated from Eq. (12) and a similar procedure is used in the next time interval.

Let us consider the results of our study of how the error of approximation of the heat flux acting on the surfaces Ω_1 and Ω_2 between elements of a composite structure affects the error of TCR determination.

As the values of the temperatures at the observation points for determining the TCR's we take the results from the solution of the stationary heat-conduction problem in the linear formulation, which were obtained by the method of finite differences for a rectangular region with a 2:2 ratio of sides, approximated by a three-dimensional grid with spacing $h = 0.1$. When the IHC problem is solved the values of the temperature function in the vicinity of the contact boundary have an approximation accuracy of the order of $O(h)$ and the results from the solution of this same problem in the linear formulation under conditions of an ideal contact will have an accuracy $O(h^2)$. In order to increase the accuracy of the approximation of the temperature function when solving the IHC problem, we decrease the approximation step near the contact boundary (on either side) to 0.1 h. The value of this step was established by means of a computational experiment, consisting in determining the heat flux and the surface temperature at the contact boundary between two rectangular regions Ω_1 and Ω_2 under the conditions of ideal contact. It was expressed as the equality of the temperatures at the contact surface with the aid of the spectral functions of the heat flux for both regions and the reactions of the known boundary effects $T_{\Omega_1}(x_s,\ 1)$ and $T_{\Omega_2}(x_s,\ 1)$ for each approximation node at the contact boundary:

$$T_{\Omega_1}(x_s,\ 1) + \sum_{j=0}^{m} b_{5j} W_{5j}(x_s,\ 1)\big|_{\Omega_1} = T_{\Omega_2}(x_s,\ 1) + \sum_{j=0}^{m} b_{5j} W_{5j}(x_s,\ 1)\big|_{\Omega_2}, \tag{17}$$

where s is the number of the nodal point with respect to the x coordinate. By solving the system of linear algebraic equations (17) we determined the heat-flux approximation parameters b_{5j} and then calculated the surface temperature, which was compared with the corresponding temperature obtained by solving the problem for the entire region Ω. The root-mean-square deviations between the results of the solutions on the contact line was less than $0.001\ T_{max}$.

Figure 1 shows the results of TCR approximation when the heat flux $q_5(x, 1)$ was approximated with a polynomial of the second degree (curves 1-6) and fourth degree (curves 7-9). As the base for comparison of the TCR's for R = 1 we took the value corresponding to the thermal resistance of the body contained between the contact surface and the surface on which the observation points are located. In determining the TCR's we varied the number of observation points, for which the values of the temperature were taken from the solution of the direct problem.

We give the coordinates of the observation points used in the calculations: $P_1(0.1, 1.1)$, $P_2(0.5, 1.1)$, $P_3(1.0, 1.1)$, $P_4(1.5, 1.1)$, $P_5(1.9, 1.1)$, $P_6(0.1, 0.9)$, $P_7(0.5, 0.9)$, $P_8(1.0, 0.9)$, $P_9(1.5, 0.9)$, and $P_{10}(1.9, 0.9)$.

The notation of the curves in Fig. 1 corresponds to the variants of the solutions of the IHC problem with the observation points arranged as follows: curve 1) P_1, P_2, P_5; 2) P_1-P_5; 3) 19 points on the coordinate line y = 1.1; 4) P_1, P_3, P_5, P_6, P_8, P_{10}; 5) P_1-P_{10}; 6) 19 points each on coordinate lines y = 1.1 and y = 0.9; 7) P_1-P_5; 8) P_6-P_{10}; 9) P_1-P_{10}.

Since the temperatures at the observation points have been taken from the solution of the direct heat-conduction problem under the conditions of ideal contact between regions Ω_1 and Ω_2, the distributed TRC's obtained can be considered to be the approximation errors of the boundary condition.

Upon analyzing Fig. 1, we can distinguish the following distinctive features that accompany solution of the IHC problem.

If the observation points are located in the hotter element, the approximation error in the TRC determination is predominantly positive (see curves 1-3 and 7).

When the observation points are located in the cooler element the TRC approximation error is negative (see curve 8).

If the observation points are in both elements and are arranged in pairs opposite each other on either side of the contact surface, the range of the TRC approximation error decreases in comparison with the results obtained by solving the problem when the observation points are located in one of the elements (see curves 1 and 4, 2 and 5, 7 and 9).

For a fixed degree of the polynomial approximating the heat flux $q_5(x, 1)$ there exists a limit to the number of observed points, after which the approximation error virtually does not change (see curves 1, 2, and 3, curves 5 and 6, and curves 7-9, which differ only slightly from the results when a large number of observation points is used). The degree of the approximating polynomial affects primarily the temperature difference between the surfaces in contact. As the degree of the polynomial increases the temperature difference decreases, which means that the TRC approximation error also decreases. The degree of the polynomial can increase to a certain limit, after which any further increase is undesirable. Since the approximating capabilities of the polynomial function have been virtually exhausted, [it turned out that for the methodological problem under consideration the fourth degree is the limit for the polynomial approximating $q_5(x, 1)$. In our study the degree of the polynomial varied from the second to the sixth, inclusively].

The size of the step of three-dimensional approximation near the contact boundary has an effect mainly on the error of the method; as it decreases the accuracy of the results increases while the approximation error for the boundary condition changes little in the direction of a decrease.

Our studies have made it possible to evaluate the level of the expected errors of the method in the TRC determination. The inverse problems are sensitive to a variety of errors, including errors in the approximation of the heat equations. When solving the problems, therefore, a dense grid must be used in the region of the contact of elements of the composite body.

If the ratio of the thermal conductivities of the materials in contact is small, we can easily reconcile the equality of the temperature drops per approximation step in space by changing it on one side of the contact boundary for the body with the lower thermal conductivity, i.e., by using an adaptive irregular gird. When the themal conductivity ratio is larger the use of an irregular grid is subject to limitations due to the growth of the system of finite-difference equations, which cannot always be solved with the available computers. Moreover, the construction of an adaptive grid requires a lot of computer time.

The application of the Kirchhoff approximation to the initial mathematical model allows us to solve the problem on a regular grid, since the boundary conditions (6) are transformed into the equality of the derivatives of the new functions with respect to the normal. In this case at the point of contact there is a step between the new functions, which increases as the ratio of the thermal conductivities of the materials in contact increases. The IHC problem is best solved (from the standpoint of computational difficulties) by using the method when the observation points are arranged in one of th ebodies in contact.

In summary, the proposed method of solving the IHC problem on the basis of limited information about the thermal state of the composite body allows the TRC to be calculated with engineering accuracy by numerical methods, the use of which is necessitated, as a rule, by the complex geometry of the objects studied. The most promising for solving problems of this class is the regional-structural method [2], which allows the heat flux to be determined in a continuous form.

NOTATION

Here T denotes the temperature; x and y are the spatial coordinates; t is the time coordinate; q is the heat flux, a_{ij} and b_{ij} are the parameters of the boundary-effect functions; W_{ij} are the spectral boundary-effect fucntions; c_v is the specific heat at constant volume; λ is the thermal conductivity; α is the heat-exchange coefficient; and R is the thermal contact resistance. Indices: s is the number of the observation point; sur denotes surface; and m denotes medium.

LITERATURE CITED

1. Yu. M. Matsevityi, A. P. Slesarenko, and O. S. Tsakanyan, Inzh. Fiz. Zh., 53, No. 3, 480 (1987).
2. A. O. Slesarenko, "Development of an algebra-logic method and its application to multidimensional nonlinear heat-conduction problems for homogeneous and composite media," [in Russian], Author's Abstract of Doctoral Dissertation, Moscow (1984).

APPLICATION OF DIRICHLET AND NEUMANN PROBLEMS IN CONNECTION WITH STUDIES OF NONSTATIONARY HEAT CONDUCTION

V. P. Kozlov, V. S. Adamchuk, and V. N. Lipovtsev

UDC 536.21

Regularities in the development of three-dimensional nonstationary temperature fields in semi-bounded iso- and orthotropic media are deduced under discontinuous boundary conditions of the first or second kind, given in the most general form.

A theoretical foundation for the creation of modern methods and measuring tools for the nondestructive control of thermophysical characteristics (TPC) of various materials is furnished by appropriate solutions of many-dimensional nonstationary problems of heat conduction with discontinuous boundary conditions (BC) [1-22]. As a result of the action of arbitrary discontinuous BC on surfaces of a medium being investigated, temperature fields arising directly from a boundary surface (in a region of action of discontinuous BC) will carry thermophysical information concerning the whole complex of TPC for the given medium. This latter circumstance makes it possible to organize complex thermophysical measurements of various materials without invading its intrinsic structure (the so-called methods and means of nondestructive control of TPC).

In the present paper we consdier the classical formulations of Dirichlet and Neumann problems in a nonstationary version and apply them to the solution of corresponding axially-

Gosplan BSSR Scientific-Research Economics Institute, Minsk.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 58, No. 1, pp. 141-145, January, 1990. Original article submitted June 7, 1988.